# Calibration of Time Domain Reflectometry for Water Content Measurement Using a Composite Dielectric Approach

KURT ROTH, RAINER SCHULIN, HANNES FLÜHLER, AND WERNER ATTINGER

Institute of Terrestrial Ecology, Soil Physics, Swiss Federal Institute of Technology, Zürich

Time domain reflectometry (TDR) has been developed to an operational level for the measurement of soil water content during the past decade. Because it is able to provide fast, precise and nondestructive in situ measurements, it has become an alternative to the neutron scattering method, in particular for monitoring water content under field conditions. One of the major disadvantages of the neutron scattering technique is that, due to the relatively high sensitivity of the signal to factors other than water content, site-specific calibration is usually required. In this paper a calibration curve for the TDR method is presented which is not restricted to specific soil conditions. The calibration is based on the dielectric mixing model of Dobson et al. (1985). Measurements of volumetric water content and dielectric number at eleven different field sites representing a wide range of soil types were used to determine the parameter of the model by weighted nonlinear regression. The uncertainty (root mean square error) of water content values calculated with the optimized calibration curve was estimated not to exceed 0.013 cm<sup>3</sup>/cm<sup>3</sup>. This value is comparable to the precision of the thermogravimetric method. From a sensitivity analysis it was determined that the temperature dependence of the TDR signal may have to be corrected to obtain optimum accuracy.

### INTRODUCTION

Soil water content is a key variable in agricultural water management and hydrological modeling, as well as for most types of soil studies in general. Monitoring the volumetric water content in the field calls for a fast and sufficiently accurate method which allows one to take repetitive measurements at the same location. The standard method of measuring the volumetric water content of a soil sample is the thermogravimetric method which consists of determining the weight loss after a specified time of oven drying at 105°C and relating it to the volume. This method is time consuming and destructive to the sampled soil and therefore cannot be used for repetitive measurements at exactly the same location. It is still indispensable, however, as a standard method or calibration purposes. The most widely used field techpiques for repetitive in situ measurements of the volumetric water content are the neutron moderation and the gamma attenuation techniques. These methods are nondestructive, except for the initial installation of access tubes, and comparatively fast. Both methods, however, involve a radiation hazard and thus require heavy protection shields. Moreover, peutron probes require soil-specific calibration, whereas ordinary field gamma probes are relatively imprecise. Another important disadvantage of the neutron scattering probe that the sampling volume strongly depends on the water content itself.

Most of the major disadvantages of the radiation techniques do not arise with methods in which water content is etermined from the dielectric properties of wet soils. (The electric number, or electric permittivity, is usually defined a complex entity. However, in this paper, "dielectric number" refers to the real part only. The imaginary part of dielectric number is related to the electrical conductivity

of the soil and may be used to study soil salinity [Dalton and van Genuchten, 1986].) Since the dielectric number of a material is frequency-dependent, the sensitivity of these methods is also frequency-dependent. The most sensitive frequency range for soil water content determination from measurements of the dielectric number of wet soils lies approximately between 50 MHz and 10 GHz. At 1 GHz and 20°C the dielectric number is 80.36 for water, 3-5 for major soil minerals, and 1 for air [Handbook of Physics and Chemistry, 1986]. This large disparity of the dielectric numbers of water and other soil constituents makes dielectric methods insensitive to soil composition and texture. At frequencies below 50 MHz the dielectric number of moist soil depends strongly on soil type [Smith-Rose, 1935]. At frequencies above 10 GHz it falls off due to water relaxation [Hoekstra and Delaney, 1974]. Among the methods operating in this most sensitive frequency range are the time domain reflectometry method (TDR) [Davis, 1975; Topp et al., 1980; Dasberg and Dalton, 1985] and free space transmission techniques (radar) [Dobson et al., 1985; Hallikainen et al., 1985].

Two different approaches have been used to relate soil water content and dielectric number for the calibration of the TDR method. In the first approach, functional relationships are selected purely by their mathematical flexibility to fit the experimental data points. No attempt is made to give a physical justification. This approach was taken by Wobschall [1977] who used a second-order polynomial to relate the high-frequency limit of the dielectric number with volumetric water content, soil porosity, dielectric number of the solid phase, micropore volume, and fraction of dispersed (noncontinuous) water. Later, Topp et al. [1980], working below the relaxation frequency of water, determined a third-order polynomial relationship between dielectric number  $\varepsilon_c$  and volumetric water content  $\theta$ ,

$$\theta = -5.3 \times 10^{-2} + 2.92 \times 10^{-2} \varepsilon_c - 5.5 \times 10^{-4} \varepsilon_c^2$$

Now at Department of Soil and Environmental Sciences, Uni-

 $+4.3 \times 10^{-6} \varepsilon_n^3$  (1)

Opyright 1990 by the American Geophysical Union.

for which they gave an error estimate of 0.013 for  $\theta$ . The main advantage of this calibration equation is that it does not require the determination of any additional soil parameters. However, its validity has not been demonstrated for the whole range of possible water contents and porosities.

In the second approach the functional form of the calibration equation is derived from dielectric mixing models which relate the composite dielectric number of a multiphase mixture to the dielectric numbers and volume fractions of its constituents, based on assumptions about the geometrical arrangement of the constituents [e.g., Tinga et al., 1973]. In the case of a perfectly layered two-phase medium (with dielectric numbers  $\varepsilon_1$  and  $\varepsilon_2$  and volume fractions  $\nu_1$  and  $\nu_2$  of the two phases, respectively) the composite dielectric number  $\varepsilon_c$  is

$$\varepsilon_c = (\nu_1 \varepsilon_1^{\alpha} + \nu_2 c_2^{\alpha})^{1/\alpha} \tag{2}$$

where  $\alpha=1$  if the electric field is parallel to the layering, and  $\alpha=-1$  if the field is perpendicular to the layers [Brown, 1956]. Birchak et al. [1974] found  $\alpha=0.5$  for an isotropic two-phase medium from travel time calculations for electromagnetic waves. Ansoult et al. [1984] obtained (2) using a discretized statistical model for a wet soil. Their analysis, in which they related  $\varepsilon_c(\theta)$  to the degrees of freedom of electric charge mobility, resulted in  $\alpha=+1$  and  $\alpha=-1$  for limiting cases of their model. The parameter  $\alpha$  thus summarizes the geometry of the medium with relation to the applied electric field.

The mixing law given by (2) may be extended to a three-phase system to describe wet soil:

$$\varepsilon_{c} = (\theta \varepsilon_{w}^{\alpha} + (1 - \eta) \varepsilon_{s}^{\alpha} + (\eta - \theta) \varepsilon_{a}^{\alpha})^{1/\alpha}$$
 (3)

where  $\eta$  is the soil's porosity,  $1 - \eta$ ,  $\theta$ , and  $\eta - \theta$  are the volume fractions, and  $\varepsilon_s$ ,  $\varepsilon_w$ , and  $\varepsilon_a$  are the dielectric numbers of the solid, aqueous, and gaseous phase, respectively. Following Birchak et al. [1974], Alharthi and Lange [1987] assumed  $\alpha = 0.5$ . After introducing adsorbed water as a fourth phase, Dobson et al. [1985] determined  $\alpha$  by regression from data for different frequencies (1.4-18 GHz) and soil types (ranging from sandy loam to silty clay) and obtained  $\alpha = 0.65$ .

In this study the mixing law approach, equation (3) with the parameter  $\alpha$  to be determined, is used to obtain a single calibration curve whose validity is not restricted to laboratory conditions, specific soil types, or to a limited water content range. In situ measurements of the soil's dielectric number and of the thermogravimetric water content at various field sites are used to determine  $\alpha$  by a weighted nonlinear regression. The uncertainty of the optimized value of  $\alpha$  is evaluated using Monte-Carlo simulation. The uncertainty of water content calculations on the basis of this calibration is estimated using error propagation analysis. (In this paper, the term "uncertainty" refers to the "square root of estimation variance.")

#### EXPERIMENTAL PROCEDURE

The application of the TDR method to measurements in soils has been described in detail, among others, by *Topp et al.* [1980] and *Dalton and van Genuchten* [1986]. It requires the installation into the soil of two wires or rods to act as a parallel transmission line. For measurement, steplike elec-

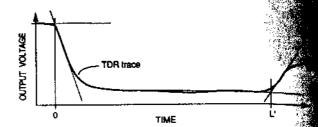


Fig. 1. Evaluation of a typical TDR trace. The time it takes pulse to move down the TDR probe and back is proportional to distance L' between the intersections of two tangents, as indicated by the dotted lines.

tromagnetic signals are transmitted into the soil along a guide. When the signal has traveled to the end of the guid it is reflected back to the instrument where its return recorded. The velocity of an electromagnetic wave in medium with relative dielectric number  $\varepsilon$  and relative mannetic permeability  $\mu$ , both relative to vacuum, is given by

$$c = \frac{1}{(\varepsilon \mu)^{1/2}} c_0$$

where  $c_0$  is the speed of light in vacuum [Jackson, 1975]. It magnetic permeability of soils usually equals unity, sin they rarely contain significant amounts of ferromagnet components. Therefore the dielectric number of the soil components be calculated directly from the travel time t of the Tosignal and the length l of the transmission line as

$$\varepsilon = \frac{t^2}{\mu} \left( \frac{c_0}{2l} \right)^2$$

The TDR probe used in this study consisted of two parallesteel rods which had a diameter of 0.6 cm, a length of 30 cand a distance of 4.8 cm from each other. The rods were he by a plexiglas handle, where they were soldered to symmetric, shielded  $185 \Omega$  cable (Belden 9090). The cable was connected to the portable Tektronix TDR Cable Tester 1502 through an impedance matching transformer (Anzac TP103).

The recorded signals were analyzed manually from paper printout. Figure 1 shows a typical TDR signal. The length denoted by L' represents the travel time of the reflected signal forth and back along the probe. This travel time includes signal propagation within the handle which not inserted into the soil. To compensate for the influence of the handle, all signal length readings L' were corrected by subtraction of a constant offset length L". This offset length was determined by linear regression on measurements pure water using various rod lengths. The corrected readings were divided through a (offset-corrected) reference signal length  $L_w$  determined for pure water at a temperature of 20°C. (This unconventional referencing was chosen to get measurements that do not depend on the instrument and in calibration.) Given proportionality between travel time t and corrected signal length L, it follows from (5) that the ratio between the composite dielectric number of soil,  $\varepsilon_c$ , and the dielectric number of water at 20°C, ε<sub>w</sub>(20), can be calculated from the relative signal length  $L/L_w$  as

TABLE 1. Description of Soils Used for Measurements

Location	Apparent Density, kg m <sup>-3</sup>	Clay, % wt	Silt, % wt	Sand, % wt	Organic Carbon, % wt	Soil Type* (U.S. Soil Taxonomy), % wt
Abist†	1.43	21.2	30.1	48.7	1.27	Aquic Eutrochrept
Buchberg†	1.08	14.5	47.5	38.0	0.9	Aquic Arenic Hapludalf
Chnoden† (Bt)	0.43	54.5	41.2	4.3	13.5	Mollic Haplaquept
Chnoden† (Gr)	0.85	43.6	45.5	10.9	3.0	Mollic Haplaquept
Gamperfin‡	0.12	2.0	0.0	0.0	98.0	Typic Sphagnofibrist
Höri	1.22	21.2	34,4	44.4	3.0	Arenic Hapludalf
Laufenburg*	1.38	9.4	19.0	71.6		Umbric Distrochrept
Oberforst† (Ah)	1.28	15.6	60.1	24.3	1.0	Typic Hapludalf
Oberforst† (Bt)	1.44	21.3	59.3	19.4	0.4	Typic Hapludalf
Wallisellen	1.34	17.2	38.8	44.0	2.4	Typic Hapludalf
Winzlerboden†	1.13	9.1	9.6	82.3	3.3	Typic Dystrochrept- Arenic Hapludalf
Zürichberg (Fi)	1.03	33.2	35.8	31.0	4.4	Aquic Hapludalf
Zürichberg (Bu)	1.12	23.5	37.1	39.4	4.1	Aquic Hapludalf

<sup>\*</sup>P. Lüscher, personal communication, 1989.

$$\zeta_c = \frac{\varepsilon_c}{\varepsilon_w(20)} = \left(\frac{L}{L_w}\right)^2 \tag{6}$$

For the calibration performed in this study, TDR field measurements and soil samples for thermogravimetric determination of water content were taken at 11 different sites in the surroundings of Zürich, Switzerland, representing a range of different soil types, as listed in Table 1. Measurements were only taken once at each site. The volumetric water content varied between  $\theta_v = 0.08$  for the sandy Winzlerboden soil and  $\theta_v = 0.92$  for the Gamperfin peat bog.

At each site, except for the peat bog, a small profile was dug to enable horizontal installation of the TDR probe at desired depths. After recording TDR signals and soil temperature the soil above the TDR probe was carefully removed until the probe was covered by only 6 cm of soil. After removing the probe, three volumetric soil samples were taken using steel cylinders (ID, 10.0 cm; height, 12.7 cm) which were driven vertically into the remaining soil, thus sampling approximately the same volume as with the TDR. The samples were dried at 105°C for 48 hours. In addition to the volumetric water content, bulk density and porosity were determined. Porosity was calculated from measured bulk densities and real densities that were measured previously at the same sites.

For the peat bog, TDR measurements were not taken in situ, but in large undisturbed soil cores in the laboratory. The cores were originally taken by polyvinyl chloride cylinders (ID, 19.5 cm; height, 48.0 cm) which were pushed vertically into the soil, excavated, and removed after the sample had been cut from the underlying material. The sealed samples were stored until use in a horizontal position a cooling chamber at 5°C to slow down microbial growth. The TDR probe was installed into the center of the cylinder. After the TDR measurement the entire cylinder was cut into stall pieces and oven dried for several days for gravimetric enter content determination. A drying temperature of 65°C chosen to prevent thermal decomposition of the organic latter. Porosity was estimated to be 0.95.

#### ANALYSIS OF THE RESULTS

Figure 2 shows a scatter plot of the measured relative soil dielectric number  $\zeta_c$ , as defined in (6), versus measured volumetric water content. The length of the horizontal and vertical bars represent the standard deviation of replicate measurements of water content and dielectric number for each sample. The light gray curve represents the calibration curve of *Topp et al.* [1980], as calculated for  $\zeta_c$ , with  $\varepsilon_w = 80.36$  and  $\varepsilon_c$  from (1). It gives fair predictions in the range of water contents up to 0.5 but considerable deviations for the

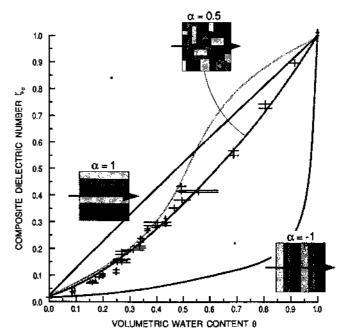


Fig. 2. Measured composite dielectric number  $\zeta_c$  as a function of volumetric water content  $\theta$  and different empirical models describing this relation. The error bars represent estimated measurement uncertainties. The light gray curve is calculated from (1) and the dark gray curves from (7), with extreme values of the parameter.

<sup>†</sup>Richard et al. [1981].

<sup>‡</sup>M. Schneebeli, personal communication, 1989.

wet soils above that limit. The dark curves were calculated for the mixing model (equation (3)). Relating the dielectric number not to vacuum but to pure water at 20°C, (3) was used here in the analogous formulation

$$\zeta_c = (\theta \zeta_w^{\alpha} + (1 - \eta) \zeta_s^{\alpha} + (\eta - \theta) \zeta_w^{\alpha})^{1/\alpha}$$
 (7)

where  $\zeta_w = \varepsilon_w/\varepsilon_w(20)$ ,  $\zeta_s = \varepsilon_s/\varepsilon_w(20)$ , and  $\zeta_a = \varepsilon_a/\varepsilon_w(20)$  are the dielectric numbers relative to water at 20°C of the solid, aqueous, and gaseous soil phase, respectively.

Values for the dielectric numbers of the three phases were taken from the literature. For the solid and the gaseous phase, temperature dependence of dielectric number was assumed to be negligible. Taking  $\varepsilon$  values of 1.0 for soil air, 3.9 for the solid phase of mineral soils, 5.0 for the solid phase of organic soils, and 80.36 for the reference state of water at 20°C, the resulting  $\zeta$  values were  $\zeta_a = 0.012$ ,  $\zeta_s = 0.048$  for mineral soil, and  $\zeta_s = 0.060$  for organic soil. For the aqueous phase, solute effects in the soils under study were considered to be negligible. However, temperature dependence was taken into account, assuming

$$\varepsilon_w(T) = 78.54[1 - 4.579 \times 10^{-3}(T - 25)]$$

$$+1.19 \times 10^{-5} (T-25)^2 - 2.8 \times 10^{-8} (T-25)^3$$
 (8)

where T is temperature in degrees Celsius [Handbook of Physics and Chemistry, 1986].

Given the volumetric fractions and the dielectric numbers of the three phases, the composite dielectric number depends only on the geometry parameter  $\alpha$  of the mixing model given by (7). To illustrate the range of variation, Figure 2 shows model curves of the  $\zeta_c(\theta)$  relationship for three different  $\alpha$  values. These curves were calculated for a constant temperature of 15°C, and the following relationship between porosity and water content:

$$\eta(\theta) = 0.710 - 1.868\theta + 3.954\theta^2 - 1.796\theta^3 \tag{9}$$

The latter relationship was obtained by fitting a third-order polynomial to measured water content and porosity data. (It was determined exclusively for the illustrative purpose of showing continuous model curves which have significance to the measured data points shown in Figure 2. Since the water content measurements in this study represent just a single realization sampled at random out of a continuous range of possible moisture conditions for each soil, this relationship cannot claim any further physical meaning.) The two curves for the mixing model shown in Figure 2 were calculated for the limiting values of  $\alpha$  (+1 and -1). Whereas the measured data points fall within the range outlined by these curves, the calibration function given by (1) leaves it above a water content of about 0.55. This indicates that extrapolation of this function leads to results which in principle are physically questionable.

With all other parameters constant, TDR calibration with the dielectric mixing model of *Dobson et al.* [1985] is tantamount to the determination of the geometry parameter  $\alpha$  in (7). This was achieved by minimizing the weighted least squares sum

$$\phi(\alpha) = \sum_{i} \frac{\left[\theta_{\text{grav}}^{i} - \theta(\zeta_{c}^{i}, \zeta_{w}^{i}, \zeta_{s}^{i}, \zeta_{a}^{i}, \eta^{i}; \alpha)\right]^{2}}{\sigma_{\text{grav}}^{i2} + \sigma_{\text{TDR}}^{i2}}$$
(10)

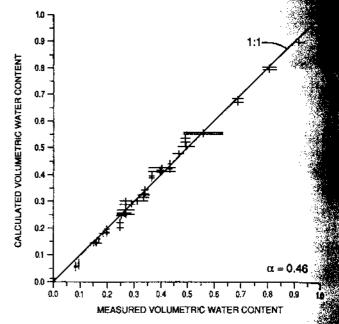


Fig. 3. Comparison between volumetric water content  $\theta$  measured thermogravimetrically and calculated from TDR measurements (equation (11), with  $\alpha = 0.46$ ). Error bars represent estimatemeasurement and modelization uncertainties, respectively.

where  $\theta_{\text{grav}}^i$  and  $\sigma_{\text{grav}}^{i2}$  denote the mean and the variance thermogravimetric water content measurements at the sampling point, respectively.  $\theta_{\text{TDR}}^i = \theta(\zeta_c^i, \zeta_w^i, \zeta_s^i, \zeta_a^i, \eta^i;$  and  $\sigma_{\text{TDR}}^{i2}$  are the calculated water content value and estimation variance, respectively.  $\theta_{\text{TDR}}^i$  is obtained from  $\eta$  as

$$\theta = \frac{\zeta_c^{\alpha} - (1 - \eta)\zeta_s^{\alpha} - \eta\zeta_a^{\alpha}}{\zeta_c^{\alpha} - \zeta_c^{\alpha}} \tag{11}$$

The variance  $\sigma_{\text{TDR}}^{i2}$  was calculated from estimated measurement errors employing the error propagation equations given below (equations (14)–(17)). The weighting factor in the denominator on the right-hand side of (10),  $\sigma_{\text{grav}}^{i2} + \sigma_{\text{TDR}}^{i2}$  was introduced to account for the fact that the variance of thermogravimetric as well as of TDR measurements varied considerably between different sampling points. Figure 3 shows a scatter plot of calculated  $\theta_{\text{TDR}}^i$  values versus measured  $\theta_{\text{grav}}^i$  values. The uncertainty of these values, as given by  $\sigma_{\text{grav}}^i$  and  $\sigma_{\text{grav}}^i$ , is represented by vertical and horizontal error bars, respectively.

Minimizing  $\phi(\alpha)$  with the Levenberg-Marquardt algorithm [Press et al., 1986], the optimum value for  $\alpha$  was found to be 0.46. The uncertainty of this value was estimated using Monte Carlo simulation. Varying the values for the observed (thermogravimetric) water content, composite dielectric number, and temperature at random, and independently of each other within a range given by the uncertainty of each parameter, and repeating the minimization procedure 1000 times, the uncertainty  $\sigma_{\alpha} = 0.007$  was found for the optimized value of  $\alpha = 0.46$ .

It was assumed that the weighted residuals of the least squares sum in (10),

$$\Delta^{i} = \frac{\theta_{\text{grav}}^{i} - \theta(\zeta_{c}^{i}, \zeta_{w}^{i}, \zeta_{s}^{i}, \zeta_{a}^{i}, \eta^{i}; \alpha)}{(\sigma_{\text{grav}}^{i2} + \sigma_{\text{TDR}}^{i2})^{1/2}}$$
(12)

variance. Then  $\phi(\alpha)$ , as defined by (10), has  $\chi^2$  statistics with N-1 degrees of freedom [Lindgren, 1976], and with increasing sample size N, therefore asymptotically approaches a normal distribution with mean  $\mu_{\phi} = N-1$  and variance  $\sigma_{\phi}^2 = 2(N-1)$ .

In total, 51 measurements were taken. The sum in (10) thus consists of N=51 terms, and the expected minimal value of  $\phi(\alpha)$  is  $\mu_{\phi}=50$  with a standard deviation  $\sigma_{\phi}^2=10$ . The actual value calculated with (10) was  $\mu_{\phi}=57.2$ . This indicates that errors due to inappropriate choice of the model were not significant relative to the accuracy of the measurements.

### ERROR PROPAGATION ANALYSIS

by TDR, calculation of soil water content with (11) requires several other parameters to be determined which may vary significantly between different locations, such as porosity, temperature, or dielectric number of the solid matrix. Supposing that errors in the determination of these parameters are mutually uncorrelated, the estimation variance of water content predictions is given in a first-order approximation by

$$s_{\theta}^{2} = \sum_{j} \left(\frac{\partial \theta}{\partial p_{j}}\right)^{2} s_{p_{j}}^{2} \tag{13}$$

where  $p_j$  stands for the jth parameter of (11) and  $s_{p_j}^2$  for its estimation variance. Partial differentiation of  $\theta$  with respect to the other variables in (11) yields the following sensitivities:

$$\frac{\partial \theta}{\partial \zeta_c} = \frac{\alpha \zeta_c^{\alpha - 1}}{\zeta_\omega^{\alpha} - \zeta_a^{\alpha}} \tag{14}$$

$$\frac{\partial \theta}{\partial T} = -\frac{\theta \alpha \zeta_w^{\alpha - 1}}{\zeta_w^{\alpha} - \zeta_a^{\alpha}} \frac{d\zeta_w}{dT} \approx \frac{\theta \alpha \zeta_w^{\alpha - 1}}{\zeta_w^{\alpha} - \zeta_a^{\alpha}} 4.5 \times 10^{-3}$$
 (15)

$$\frac{\partial \theta}{\partial \zeta_s} = \frac{\alpha (1 - \eta) \zeta_s^{\alpha - 1}}{\zeta_w^{\alpha} - \zeta_a^{\alpha}} \tag{16}$$

$$\frac{\partial \theta}{\partial \eta} = \frac{\zeta_s^{\alpha} - \zeta_a^{\alpha}}{\zeta_w^{\alpha} - \zeta_a^{\alpha}} \tag{17}$$

$$\frac{1}{\zeta_c^{\alpha} - \zeta_a^{\alpha}} \left\{ \zeta_c^{\alpha} \ln (\zeta_c) - (1 - \eta) \zeta_s^{\alpha} \ln (\zeta_s) - \eta \zeta_a^{\alpha} \ln (\zeta_a) \right\}$$

$$-\theta[\zeta_w^{\alpha} \ln(\zeta_w) - \zeta_a^{\alpha} \ln(\zeta_a)]\} \qquad (18)$$

the right-hand side of (15), equation (8) was used to existing  $\zeta_w$ , as defined in conjunction with (7), neglecting the conjunction with (7), neglecting the confidence of the conjunction with (7), neglecting the conjunction with (7), neglecting the conjunction with (7), neglecting the conjunction of the conjunction with (7), neglecting the conjunction with (7), neglecti

$$\frac{s_{\zeta_c}^2}{\zeta_c^2} = 4 \left( \frac{s_L^2}{L^2} + \frac{s_{L_w}^2}{L_w^2} \right) \tag{19}$$

here  $s_L^2$  and  $s_{L_s}^2$  are the estimation variances for the TDR mat lengths. Variations of the dielectric number of soil air to variable composition as well as sensitivity to such

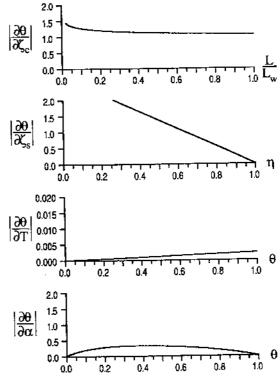


Fig. 4. Sensitivity of measured volumetric water content to different parameters. The vertical axis of the third plot is stretched by a factor of 100 compared to the others.

variability are very small and are therefore not explicitly considered in this analysis. Likewise, the composition of the soil solution was considered to be a negligible source of uncertainty for nonsaline soils. In Figure 4 the absolute value of the sensitivities is plotted as a function of the most important variable. The partial uncertainty  $s_{\theta,p_j}$  of calculated water content caused by a measurement error of parameter  $p_j$  depends on the uncertainty  $s_{p_j}$  of the parameter and on the sensitivity of calculated water content on this parameter. As the error  $s_{p_j}$  is determined by the measurement procedure, nothing general can be said about its magnitude. Table 2

TABLE 2. Uncertainty of Calculated Volumetrie Water Content θ due to Hypothetical Measurement Uncertainities for Two Extreme Soil Types

		Dry	Wet
Soil parameters	θ	0.08	0.93
Son parameters	ž.	0.06	0.88
	ζ, Τ, ℃	20.	20.
	ζ, _	0.048	0.048
	ກ	0.4	0.95
Assumed uncertainties	SZ	0.003	0.015
Assumed uncertainties	s <sub>T</sub> , ℃	1.0	1.0
	s <sub>ζ</sub> , –	0.005	0.005
	$s_{\eta}$	0.05	0.05
	S <sub>a</sub>	0.007	0.007
Resulting partial uncertainties	\$ 0,4	0.0072	0.0085
Resulting partial bileeramieles	S 0, 40	0.0002	0.0022
	S 9. 7	0.0082	0.0007
	S 8, 77	0.0067	0.0067
	S 0.0	0.0011	0.0006
Resulting uncertainty of $\theta$	- <del>0</del> ,2 Sg	0.013	0.011
Relative error of $\theta$	s d 0, %	16.	1.2

shows values of two sample calculations of  $s_{\theta, p_i}$  and  $s_{\theta}$ , the total uncertainty of calculated water content for a typical measuring procedure in a dry and a wet soil. In these calculations it was assumed that temperature was measured in situ, as this is cheap and fast, whereas porosity and dielectric number of soil matrix were estimated because their measurement would be too time consuming.

#### DISCUSSION AND CONCLUSIONS

The results show that the dielectric mixing model, as given by (3) with a constant parameter  $\alpha$ , is appropriate to describe the composite dielectric number of different soils. The best fit value of the parameter  $\alpha$  determined in our study is close to 0.5, the value used for theoretical reasons by *Birchak et al.* [1974] and *Alharthi and Lange* [1987]. In contrast, *Dobson et al.* [1985] found a value of 0.65. The difference might be due to the fact that these authors were working at higher frequencies and different modes of the electromagnetic field.

Different authors [Wang and Schmugge, 1978; Topp et al., 1980; Hallikainen et al., 1985] report a dependence of  $\zeta_c(\theta)$ on soil texture, which is presumably due to an interaction between the solid and liquid phases. Water is thought to form a thin film with a paracrystalline structure around the solid phase [Wang and Schmugge, 1978; Dobson et al., 1985]. Because of restricted rotational freedom of the water molecules in this film its dielectric number is lower than that of bulk water. The magnitude of this effect depends on the surface area and surface charge and thus on the texture of the soil. At the same water content this effect generally leads to a lower dielectric number for fine-textured soils, compared to coarse-textured ones. We could not detect this dependence in our experiment because the volumetric water content of field soils is strongly influenced by soil texture and cannot be changed arbitrarily. The data shown in Figures 2 and 3 thus typically come from sandy soils in the low-water content region and from clayey and peaty soils in the high-water content region. Any dependence of  $\zeta_c(\theta)$  on soil texture would therefore be lumped into the parameter  $\alpha$  and would tend to decrease it compared to the value which would be measured in a specific soil over a wide range of water contents (see Figure 2). The approach of the composite dielectric can, in principle, handle the effect of adsorbed water films, for example, by partitioning the liquid phase into two phases with different dielectric numbers [Dobson et al., 1985] or by making  $\zeta_w$  dependent on water saturation. At present, however, the data base is too small to estimate the parameters of such a model.

In our study the relationship between the dielectric number of soil and water content according to (3) was found to be stronger if temperature dependence of the liquid phase' dielectric number was taken into account. Since the dielectric numbers of air and solid phase are known to be much less sensitive to temperature than that of water, the influence of temperature on the dielectric number of wet soil decreases with water content. Topp et al. [1980] and Zagoskii et al. [1982], who were both working at comparatively low water contents (0.32 and 0.19, respectively), did not observe a significant temperature effect, whereas Hoekstra and Delaney [1974], working at higher frequencies and different modes of the electromagnetic field, found it to increase.

To calculate the volumetric water content from the mea-

sured dielectric number of wet soil, other parameters as perature, porosity, and dielectric number of soil matrix must be known. Whereas temperature can be measured with little additional effort, porosity and dielectric number of soil matrix would require laborious and destructive measure ments. In most cases these two parameters will therefore h estimated based on soil information data, causing an add tional uncertainty in the calculated value of the water content. Assuming reasonable estimation uncertainties, the magnitude of the corresponding uncertainties of the calcullated water content are summarized in Table 2 for a dry and a wet soil. It shows that an uncertainty of 0.05 of the porosits leads to an uncertainty of about 0.007 for the volumetric water content for both soils. On the other hand, the uncertainty due to the estimation of the soil dielectric number depends strongly on the water content. An uncertainty of Z. of 0.005 leads to an uncertainty of 0.008 for the volumetric water content of the dry soil and to 0.0007, an order of magnitude less, for the wet soil.

The uncertainty of the calculated water content due to the estimation of the parameters  $\eta$  and  $\zeta_s$  is of the same order as the uncertainty due to the measurement uncertainty of the TDR equipment. Therefore estimation, rather than measurement, of the two parameters is justified.

For the two hypothetical measurements summarized in Table 2 the uncertainty of volumetric water content calculated from TDR measurements does not depend strongly on the water content. This leads, however, to large relative uncertainties  $s_{\theta}/\theta$  for low water contents. Table 2 shows in relative uncertainty of 16% for the very dry soil compared to only 1.2% for the wet soil. To improve the accuracy in dry soils, three parameters must be determined more accurately the composite dielectric number  $\zeta_c$ , the dielectric number  $\zeta_c$  of the soil matrix, and the porosity  $\eta$ . Whereas the accuracy of  $\zeta_c$  may be improved comparatively easily (by using longer probes, using a more accurate TDR instrument, or calculating all dielectric numbers relative to air, instead of water), an increased accuracy of  $\zeta_s$  and  $\eta$  requires destructive measurements.

## REFERENCES

Alharthi, A., and J. Lange, Soil water saturation: Dielectric determination, Water Resour. Res., 23(4), 591-595, 1987.

Ansoult, M., L. W. de Backer, and M. Declercq, Statistical relationship between dielectric constant and water content in porous media, Soil Sci. Soc. Am. J., 48, 47-50, 1984.

Birchak, J. R., C. G. Gardner, J. E. Hipp, and J. M. Victor, Highdielectric constant microwave probes for sensing soil moisture, *Proc. IEEE*, 62(1), 93–98, 1974.

Brown, W. F., Dielectrics, in *Encyclopedia of Physics*, vol. 17, pp. 1-154, Springer-Verlag, New York, 1956.

Dalton, F. N., and M.Th. van Genuchten, The time-domain reflectometry method for measuring soil water content and salinity, Geoderma, 38, 237-250, 1986.

Dasberg, S., and F. N. Daiton, Time-domain reflectometry field measurements of soil water content and electrical conductivity, Soil Sci. Soc. Am. J., 49, 293-297, 1985.

Davis, J. L., Relative permittivity measurement of a sand and clay soil in situ, Geol. Surv. Pap. Geol. Surv. Can., 75-1C, 361-365, 1975.

Dobson, M. C., F. T. Ulaby, M. T. Hallikainen, and M. A. El-Rayes, Microwave dielectric behavior of wet soil, II, Dielectric mixing models, *IEEE Trans. Geosci. Remote Sens.*, GE-23(1), 35-46, 1985.

Hallikainen, M. T., F. T. Ulaby, M. C. Dobson, M. A. El-Rayes, and L.-K. Wu, Microwave dielectric behavior of wet soil, Is

- Empirical models and experimental observations, IEEE Trans. Geosci. Remote Sens., GE-23(1), 25-34, 1985.
- Handbook of Physics and Chemistry, 67th ed., CRC Press, Boca Raton, Fla., 1986.
- Hoekstra, P., and A. Delaney, Dielectric properties of soils at UHF and microwave frequencies, J. Geophys. Res., 79(11), 1699-1708, 1974.
- Jackson, J. D., Classical Electrodynamics, 2nd ed., John Wiley, New York, 1975.
- Lindgren, B. W., Statistical Theory, 3rd ed., Macmillan, New York, 1976.
- Press, W. H., B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, Numerical Recipes, The Art of Scientific Computing, Cambridge University Press, New York, 1986.
- Richard, F., P. Lüscher, and T. Strobel, Physikalische Eigenschaften von Böden in der Schweiz, vol. 1, 2, 3, 1981.
- Smith-Rose, R. L., The electric properties of soil at frequencies up to 100 MHz, Proc. Phys. Soc. London, 47, 923, 1935.
- Tinga, W. R., W. A. G. Voss, and D. F. Blossey, Generalized approach to multiphase dielectric mixture theory, J. Appl. Phys., 44(9), 3897-3902, 1973.
- Topp, G. C., J. L. Davis, and A. P. Annan, Electromagnetic

- determination of soil water content: Measurement in coaxial transmission lines, Water Resour. Res., 16(3), 574-582, 1980.
- Wang, J. R., and T. J. Schmugge, An empirical model for the complex dielectric permittivity of soils as a function of water content, NASA Tech. Memo. 79659, 35 pp., Goddard Space Flight Cent., Greenbelt, Md., 1978.
- Wobschall, D., A theory of the complex dielectric permittivity of soil containing water: The semidisperse model, *IEEE Trans. Geosci. Electron.*, *GE-15*(1), 49-58, 1977.
- Zagoskii, V. V., V. M. Nesterov, E. A. Zamotrinskaya, and T. G. Mikhailova, Dependence of dielectric permittivity of moist disperse materials on the temperature, Sov. Phys. J., 25(1), 62-65, 1982.
- W. Attinger, H. Flühler, and R. Schulin, Soil Physics, Swiss Federal Institute of Technology, ETH-Zentrum, CH-8092, Zürich, Switzerland.
- K. Roth, Department of Soil and Environmental Sciences, University of California, Riverside, CA 92502.

(Received June 26, 1989; revised April 27, 1990; accepted May 21, 1990.)